

PERSPECTIVE

# Mathematical Approaches in Studying the Ideal Image of the Goal

Alexander G. Kruglov, PhD, ScD; Alexander Yu. Vasiliev, PhD, ScD

*Moscow State University of Medicine and Dentistry  
Moscow, Russian Federation*

## Abstract

The article outlines the possible approaches in the mathematical computations of *integrated behavioral units in functional systems supporting homeostasis through in behavioral changes*. By an imbalance in the homeostasis system which initiates adaptive behavior we assume: for metabolism – a departure of the parameters from the “normal zone” to the level of a suprathreshold sensitivity of the receptors; for structures of the psychological and social spectra – to the “cognized-not cognized”, “acceptable-not acceptable” levels. For the system analysis of goal-directed behavior dynamics, we present a combination of the “creation – retention” of the ideal image of the goal and the entire effector structure of the integrated behavioral unit by introducing an integrating term, motivational gradient. The integrated Behavioral Unit (BU) is described as a psychophysiological metamer in behavioral continuum, including a mathematical description of the BU as a whole including its elements viz., the ideal image of the goal and the motivational gradient. The hemodynamic equivalent of the motivational gradient (the scalar gradient) and subjective time (the time marker) are used as the BU markers. For the mathematical description, we use the mathematical apparatus of topological spaces and elements of the string theory to open up opportunities for new approaches in psychology and neurobiology.

**Keywords:** *integrated behavioral unit; ideal image of the goal; motivational gradient; translation symmetry; scalar vector; topological space; artificial intelligence.*

## Purpose of the Work

Modern theories in psychology and neurobiology, principles and methods of experimental simulation and mathematical modeling of goal-directed behavior and computer-assisted instruction, including the approaches offered in computer-assisted instruction, lack completeness of perception and a clear description of “ideal” in the structure of goal-directed behavior. They focus on the analysis of the environment and hierarchy of goals, neural network agents, systemogenesis of neural network structures, their mathematical models, qualities of “intelligent agents” [1], etc. The core component of the integrated BU, the ideal Image of the Goal (IG) is, in our opinion, poorly interpreted in modern psychology and neurobiology.

Although we cannot furnish the IG mathematical models we are working on now in this publication, our desire

is to introduce to you some of the mathematical methods that can help to create an adaptive model of the ideal portion of BU (a metamer of behavioral continuum): viz., the ideal image of the goal.

## Discussion

According to P.K.Anokhin [2], integrated BU in the theory of functional systems of the 2<sup>nd</sup> type, which supports that homeostasis, triggered by a change in behavior, is responsible for creating, retaining and subsequently removing the ideal IG, “a space-time structure”, of the activated neural circuits of the brain. Our concept is that the afferent part of the BU is triggered by a significant imbalance in the parameter/parameters of the metabolic, psychological and social spectra, i.e. a departure from the bounds of the “normal zone”. For metabolism, a deviation from the upper/lower bounds of the “normal zone” to the level of the suprathreshold sensitivity of the receptors is involved; for structures of the psychological and social spectra involvement is to the level of “cognized-not cognized” and “acceptable-not acceptable”. The distinguishing characteristic of “living beings” is their responsiveness to any

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\*Corresponding author: Alexander G. Kruglov, PhD, ScD.  
Moscow State University of Medicine and Dentistry. Moscow,  
Russian Federation. Tel: 7-495-6110177. E-mail: [krag48@mail.ru](mailto:krag48@mail.ru)

disturbance in the homeostatic balance, to outright imbalance, i.e. the selective response to the object of an actual need. The need, being the most significant disposition of the motive, constitutes “an impelling force, a cause of the behavior” [3], which initiates the BU activation.

The “normal zone” is assumed as the aggregate of the steady performance parameters, and is created by the confidence interval boundaries, where the probability of occurrence of another parameter value is  $>95\%$ . The actual data on the average values, boundaries of the confidence interval and relationships of the basic parameters of metabolism in healthy people, obtained using the probing method for arterial and venous blood, were reported in our prior publications [4].

The methods currently available can be applied only to the assumed equivalents of BU: electrical, electrochemical, gas-hemodynamic, metabolic, behavioral, etc. It is, therefore, assumed that at the physiological level the BU core mechanism, involving the development of an acceptor of action results, i.e. IG creation and retention, is represented by an excitation circulating in a ring of interneurons [5].

Thus, the IG is generated by an activated neural circuit functioning until the actualization of the effector part of the BU is completed, until parametric equalization is reached, afference ceases and the parameter/parameters that actually caused the imbalance in the system is/are within the “normal zone”.

IG, which we perceive at the current level of understanding, is a one-dimensional, time-extended creation, an integral characteristic of the individual who stays invariably in the vector space of alternately formed goals (in simplified form) and passes through the main stages of, afference, creation of the ideal image of the goal, efference and reverse afference [2,5].

For the purpose of the system analysis of the dynamics of adaptive self-regulation, it is feasible to employ an integrating term covering the IG “creation – retention” stage and the entire effector structure of BU: the motivational gradient (MG), where  $t\text{IG} = t\text{MG}$ .

We have best defined MG as an Active Distributed System (ADS) capable of spontaneously creating, in response to the information contained in the IG, static and spatially heterogeneous structures facilitating adaptive behavior (for example, morphogenesis in a fertilized egg). ADS behavior types are described by non-linear differential equations [6].

As the MG reveals model characteristics of a future result and acts as the determinant of behavior, it establishes the mechanism and the vector of the self-regulation process resulting from goal-directed behavior producing parametric equalization, need satisfaction and motive removal. We present an equivalent MG, a hemodynamic, scalar gradient (with morphophysiological characteristics) as an MG biophysical “marker” (with the dynamics of the characteristics inherent in “correlated-correlating operating images”): venous-liquor pressure in the exchange field of actualized neural circuits [7].

Perceiving MG as the route of the fastest change in the scalar values that are changing from one space point to another, we define the “zero” (starting) point of subjective time (ST) required in the IG structure for the Expected Action Result (EAR), which in psychological time can be assumed as an abstract point, while “the present is always a certain interval”

[8]. The Expected Action Result (EAR) is a constructive outcome of the ideal image of the goal. The Action Result (AR) itself is the final stage of goal-directed behavior that is effected through the motivational gradient discontinuing the BU afference at the point of parametric equalization. The occurrence of the Expected Action Result (EAR) is the starting point of constructing an assessment scale of the time required to achieve the result. During the goal achieving process, the linear motion from MG to the AR implies the decreasing motion of the ST (to the “0” point in EAR). In other words, the decreasing time flow involves a vector which is opposite to the vectors of the effector part of the MG with increasing values of (physical, biological) time, reaching the “zero” value at the point of parametric equalization. Therefore, the ST flow can exist only if in the inward world of an individual events do exist, irrespective of their nature or type. Thus, being a one-dimensional structure, possessing a duration value ( $t\text{ST} = t\text{IG} = t\text{MG}$ ) and a vector, the ST can also find use as a vector “marker” of IG (MG).

Goal formation (voluntary/involuntary, actualized/latent, strategic/interim) can be described as the “behavioral continuum of alternately changing components” [3]: BUs “flowing” from the “previous” into the “next” (with the possibility of different sequence options, including transformation, “overlapping”, interference, etc.). The intervals between BU actualizations depend on variations in the homeostasis parameters prior to reaching the suprathreshold level and an initiation of the subsequent (latent) IG.

It is our understanding that goal formation determined by the imbalanced system is a continuous representation from one space-objective reality, to another, space-mental structures of representation (with different dimensional, time, space, etc., categories) and the subsequent projection of the mental structures (IG) into the external world through goal-directed behavior. Thus, the correlation between sets is established, where near points of the evaluation develop into near points of the values. In information interaction of two systems (either real or ideal), either of them can be understood as the model of the other system inasmuch as it represents it [9]. As the nature of IG is a one-dimensional time-extended structure that allows for omitting the distance (measurement) concept in the mental image structure, it is our perception that further computations in this field can be done by using mathematical tools of topological spaces, which make it possible to assess the proximity of the points, while ignoring the concept of distance [10]. We believe that this approach offers good opportunities for the description, study and modeling of the “modal relationships that exist between spatial patterns, coherence laws....., regardless of the relationships between the measures and values” (Listing, 1847) [11].

Integrated behavioral units are structurally identical regular sequences, which, regardless of the IG content and the end result achieved through the MG have characteristics of bijection where each element of one set correlates with an element of another set. Due to the bijection that retains the object as an information matrix in representation of the coded structure, these structures can be understood as being isomorphic [12].

During BU actualization, an equivalent, time symmetrical BU derivative structure of the hemodynamic scalar gradient, with morphophysiological parameters is present; it is a continuous representation of this gradient, a biophysical equivalent of each structurally identical regular sequence (BU), a marker of the relationship that we define as translation symmetry. Therefore, it is our understanding that the integrated behavioral unit, including the biophysical equivalent, can be defined as a psychophysiological metamer in the behavioral continuum both for tactical and strategic (sums of interval-slotted interims and continuously sequenced goals) purposes.

Significant parameter fluctuations exceeding the “normal zone” boundaries – the homeostasis “pattern” initiating MG and characterized by a totality of its own values and vectors of all the components of the space-time configuration include excited receptors, neural circuits, mental structures, metabolic systems and efferent mechanisms. In the MG dynamics, the parameters of homeostatic metabolism are actual numbers (representing continuous values), which in the homeostatic continuum constitute the predetermined nature of the dynamic scalar filled in four-dimensional space, where  $u$  (the number) =  $u(x, y, z, t)$ . Therefore, we project that it is possible to create a structural pattern corresponding to the scalar differential equation and to make a subsequent mathematical computation of the “fragments” of the psychophysiological metamer, BU having modal characteristics and existing in the dynamic scalar field. We also understand that the suggestions offered above open up new opportunities of computation of artificial intelligence models.

By defining IG and MG, we also explain the dynamics and interaction between the one-dimensional, time-extended structures generated by the fluctuating adaptive changes in homeostasis and structures in the hemodynamic biophysical equivalent, which are created by dynamic changes in the concentrations, charges, relationship mix, etc., of the physical particles and their combinations constituting the substances of metabolism. The primary interaction of the entire spectrum of these substances (with their own relationship constants) occurs under the following conditions: *sizes* –  $10^9m$ . ( $Na-0.42 \mu m$ ,  $K-133 \mu m$ ,  $H_2O-0.276 \mu m$ , etc.); *weight* –  $10^{-27}kg$ . ( $Na-22.99$ ;  $K-39.09$ ;  $H_2O-29.9$ , etc.)

## Conclusions

Having no other size parameters, barring the time parameter, the IG includes all the possible and depictable types of space and measurements, which permits drawing an assumption regarding their existence (possible building) in the mental formations of a human being where they occur in a compact (potential) condition and can be decoded (constructed) by the operating systems of the brain (thinking, imagination, etc.). For IG elements (“inward”), it acts as a space-time structure with an unlimited (unknown to us) number of levels of freedom.

The existence of the translation symmetry of the metabolic dynamics of homeostasis and the psychophysiological structures of the brain (IG, MG) makes it possible to extrapolate mathematical computations of the

markers: the scalar (hemodynamic equivalent of MG) and the vector (subjective time) markers to the core component of the metamer of the integrated behavioral unit – the ideal image of the goal.

Taking into account the one-dimensional nature of the ideal image of the goal, which has the unlimited number of levels of freedom and sets the basic characteristics for the structure, vector and behavioral continuum metamer duration, we concluded that the elements of the “string theory” (M-theory) mathematical apparatus used to compute one-dimensional extended structures setting the characteristics of weight, charge, etc. (including the dual transformation providing the switchover between the different scale systems) could be used for the mathematical description of the integrated behavioral unit, motivational gradient and the ideal image of the goal. Due to the mathematical apparatus of the basic components of the integrated behavioral unit, we will be able to identify the mathematical equivalents of the interaction mediator and relationship constant of the ideal image of the goal, using the biophysical and time “markers”. When put into practice, our suggestions (or part of them) will help to solve the problem of adequacy in the modeling of complex systems, to complement the existing research methodologies and modeling of the human psyche and artificial intelligence by using computational conclusive methods, including a mathematical analysis of the ideal mental structures.

## Competing interests

The authors declare that they have no competing interests.

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